

**ON THE NUCLEAR SHELL MODEL
AND ITS APPLICATION TO
TRANSFER REACTIONS**

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Akademisk avhandling som med vederbörligt tillstånd av tekniska fakulteten vid universitetet i Lund för vinnande av teknisk doktorsgrad kommer att offentligen försvaras å föreläsningssalen, institutionen för teoretisk fysik fredagen den 23 november 1973 kl 10.

Avhandlingen försvaras på engelska.

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This paper contains an introduction to and a summary of a dissertation based on the following publications:

- I. B. Nilsson: Inclusion of a P_4 term in the deformed shell model potential, Nuclear Physics A129 (1969) 445-468.
- II. P. Möller and B. Nilsson: On the effect of the P_6 degree of freedom in the nuclear potential at equilibrium and fission distortions, Physics Letters 31B (1970) 171-174.
- III. R.A. Broglia, S. Landowne, V. Paar, B. Nilsson, D.R. Bès and E.R. Flynn: Coupling between inelastic and pair transfer degrees of freedom: The $^{208}\text{Pb}(t,p)^{210}\text{Pb}(3^-)$ reaction analyzed in the coupled channel Born approximation, Physics Letters 36B (1971) 541-546.
- IV. R.A. Broglia, K. Kolltveit and B. Nilsson: Effective wave function calculation of the $J^\pi=0^+$ states of the Ca-isotopes through the (t,p) and (p,t) reactions, Physics Letters 37B (1971) 441-445.
- V. R.A. Broglia, P.D. Kunz and B. Nilsson: Second order pairing phase transition in the Sn isotopes: Competition between pairing and shell effects, Physics Letters 38B (1972) 383-388.
- VI. R.F. Casten, E.R. Flynn, J.D. Garrett, O. Hansen, T.J. Mulligan, D.R. Bès, R.A. Broglia and B. Nilsson: Search for (t,p) transitions to excited 0^+ states in the actinide region, Physics Letters 40B (1972) 333-337.
- VII. D.R. Bès, R.A. Broglia and B. Nilsson: Importance of the quadrupole pairing field in the $J^\pi=0^+$ vibrations of shape deformed nuclei, Physics Letters 40B (1972) 338-344.

- VIII. E.R. Flynn, G.J. Igo, R.A. Broglia, S. Landowne, V. Paar and B. Nilsson: An experimental and theoretical investigation of the structure of ^{210}Pb , Nuclear Physics A195 (1972) 97-118.
- IX. P. Kleinheinz, R.F. Casten and B. Nilsson: Hexadecapole deformation in the wolfram nuclei, Nuclear Physics A203 (1973) 539-560.
- X. T. Erikson, V. Horsfjord and B. Nilsson: On the predictions of the coexistence model for the reaction $^{42}\text{Ca}(p,t)^{40}\text{Ca}$, Physics Letters 46B (1973) 173-176.
- XI. E.R. Flynn, R.A. Broglia, R. Liotta and B. Nilsson: Elementary modes of excitation in the $^{204}\text{Pb}(t,p)^{206}\text{Pb}$ reaction, Nuclear Physics in print.

*Det du formulerer klart,
modbevises i en fart.*

*Det du maler i det dunkle,
vil som visdom laenge funkle.*

Kumbel

I. Introduction

Our understanding of nuclear structure is to a very large extent based on comparing experimental results to predictions obtained from model calculations. Among such models the nuclear shell model¹⁾ has proved indispensable. Here the basic idea is, that the nucleus is built by nucleons moving in single-particle orbits determined by an average potential generated by all the other nucleons. The single-particle orbits are then used as a basis for the diagonalisation of the remaining (relatively weak) residual interactions between the nucleons. These additional forces thus give rise to a certain distribution of the particles on the different single-particle orbits, or in other words, to certain correlations between the nucleons. It is the main task of a nuclear structure theorist to help to understand these correlations and the underlying single-particle structure.

One of the main sources of information on the one-particle properties is the one-particle transfer reaction. The basic matrix element for such a reaction is proportional to the percent to which the overlap between the nuclear states involved can be described as an one-particle orbit. The shape of the angular distribution is generally characteristic of the transferred angular

momentum. When analyzed in e.g. the distorted wave Born approximation (DWBA), such a reaction gives information about nuclear spins and spectroscopic factors which makes it possible to identify the single-particle states.

The correlations induced by the residual interactions can be tested by other reactions. In processes like (t,p) and (p,t) two neutrons are transferred in a pure relative S-state. The (t,p) reaction can thus be used to study S-state pair correlations. Accordingly it is the specific tool for studying the effects of pairing forces.

These reactions usually get the largest contribution from the process where the final state is formed directly by adding or removing nucleons from the initial state. In such cases an analysis in DWBA is useful. However, in cases where particular states are easily excited by inelastic scattering, these states occur as intermediate states in the transfer process. The corresponding multiple-step process must therefore be taken into account explicitly. This is conveniently done in a coupled channel Born approximation (CCBA) analysis.

The present paper summarises a few publications where the above mentioned concepts play a dominant role. It divides into two main parts. The first part (papers I, II and IX) deals with a few aspects of the deformed shell model, which is applied when the nuclear average field contains nonspherical components. In part two (papers III-VIII, X-XI) a number of two-nucleon transfer experiments are analyzed using different wave functions. Special attention is paid to the effects of

pairing forces. In two cases (papers IV and X) the so-called coexistence model is tested for the Ca isotopes.

II. The deformed shell model

The deformed shell model as employed by S.G. Nilsson²⁾ has had considerable success in predicting properties of nuclei having stable equilibrium deformations. The model in ref. 2 is confined to quadrupole deformations. The inclusion of higher multipoles in the potential distortion was therefore a natural extension to study.

In ref. 3 the first results of calculations using potentials containing the next even multipole (hexadecapole) deformation were reported and in paper I this extended deformed shell model was treated in some detail. The main emphasis was given to the calculation of the total energy of the nuclear system which was estimated as the sum of the Coulomb and shell energies. The Coulomb energy was calculated macroscopically with added corrections for the nuclear surface diffuseness and exchange. The shell energy was taken to be the BCS energy. In this way the total energy surface as a function of quadrupole and hexadecapole deformations was calculated for a number of actinide isotopes. In ref. 4 it was shown, that the model predicted the ground state hexadecapole deformations quite well in the rare earth region. Paper I presents corresponding predictions for the actinide region, but due to the lack of experimental data these results could not be tested. Calculations for large deformations were also performed in order to try to estimate fission barriers, but the summation method used was found to be inadequate for this problem.

This difficulty could to a large extent be resolved by the introduction of the Strutinsky method⁵⁾ for renormalizing the total shell model energy so that its smooth average behaviour as a function of deformation reproduces that of the liquid drop. Systematic calculations using this procedure were performed and applied to estimate the fission barriers in heavy and super-heavy elements in ref. 6.

Because of the considerable effects of the hexadecapole deformation, it was later found necessary to check the importance of even higher distortion multipoles. In paper II a few barriers were recalculated allowing for hexakontatettara deformations (described by the Legendre polynomial P_6). It was found, that the decrease in energy due to this distortion multipole is never more than 0.5 MeV along the path to fission. For the elements in the beginning of the actinide region the height of the fission barrier is slightly decreased or unchanged. For the heavier elements it is increased mainly due to the relatively large energy decrease at equilibrium. The comparatively small effects of the hexakontatettara deformation indicates that the potential distortion is well described and allows one to neglect higher order deformations in these calculations.

As mentioned in the introduction the specific tool to test the one-particle model with respect to the position of the single-particle levels is the one-particle transfer reaction. In paper IX such reactions are used to establish the presence of hexadecapole deformations in the wolfram isotopes. Since the neutron levels originating in the $0i_{13/2}$ spherical shell model state are the

only positive parity levels in the low energy spectrum of the odd wolfram isotopes their wave functions are dominated by the $j=13/2$ component. Therefore, the high angular momentum members of the rotational bands built on these orbits should be populated most. By using a reaction such as (τ, α) , in which transfer with large angular momentum is favoured, one gets a spectrum in which these states are easily identified. The extraction of single-particle levels from this spectrum is somewhat difficult because of strong Coriolis effects. In spite of these uncertainties, the resulting single-particle spectrum shows an unmistakable feature namely, the low K-orbits are much more compressed than a calculation using only quadrupole deformation would predict. The introduction of a hexadecapole deformation with magnitude $\epsilon_4=0.06$ produces a good fit to the data. The reason for this compression is that orbits having small or large K are located close to one of the symmetry planes. They are therefore subjected to an increasing oscillator strength when the hexadecapole deformation is made positive and consequently their energies increase. Orbits having intermediate K feel the opposite effect. The presence of hexadecapole deformation in the wolfram region is therefore verified.

III. Two-nucleon transfer and the pairing model

In open shell nuclei the pairing correlations can reach their full effect and produce a so-called superfluid system. In such a system the distinction between particles and holes is lost and the excited states (pairing vibrations⁷⁾) are built from the same single-particle subspace as is the ground state. Since all contributions

from the different configurations to the two-neutron transfer ground state cross section add with the same sign and, since, of course, the pair vibration is orthogonal to the ground state, one expects comparatively small cross sections to the pair vibrations of superfluid nuclei. On the other hand, in closed shell nuclei (like e.g. ^{208}Pb) there is a clear distinction between particles and holes. In this case both the two-particle state ($^{210}\text{Pb}(\text{gs})$) and the two-hole state ($^{206}\text{Pb}(\text{gs})$) can be excited strongly in two-nucleon stripping or pick-up from the closed shell nucleus. However, these excitations can be coupled to form a 2p-2h state in the closed shell system which is populated with cross sections similar to those of the 2p or 2h systems. Contrary to a superfluid system, a closed shell nucleus thus has two monopole states which should be seen strongly in a two-nucleon transfer experiment.

The situation for the even Sn-isotopes is somewhat intermediate to the two cases described above. The single-particle gaps are so small, that a stable superfluidity is developed. However, there are fairly large gaps for $A=114$ and $A=120$. Around these masses one should see changes in the cross sections to the first excited monopole states. The calculations presented in paper V for the cross section of ground and first excited 0^+ states in (p,t) and (t,p) reactions between the even Sn isotopes show, that the transitions near $A=114$ behave very similarly to those around a closed shell nucleus. The corresponding pattern is not seen around $A=120$. This feature is explained by the difference in the intrinsic cross section of the single configurations.

The pairing force can in fact have components of all multiplicities⁸⁾. A nucleus like ^{210}Pb which has two neutrons outside the ^{208}Pb closed shell is suitable for testing predictions of multipole-pairing forces. In papers III and VIII the reaction $^{208}\text{Pb}(t,p)^{210}\text{Pb}$ is analyzed using multipole-pairing wave functions as well as wave functions obtained from calculations using so-called realistic forces. The multipole pairing model gives as good an agreement with the data for the positive parity states as do any of the other models tested.

The low-lying octupole states present some interesting complications. There are two low-lying configurations in ^{210}Pb having spin 3^- : a two-particle state, which is mainly $(g_{9/2}, j_{15/2})3^-$, and a collective state built out of the octupole vibration in ^{208}Pb coupled to the ground state of ^{210}Pb . Taking the $^{208}\text{Pb}(3^-)$ state from an RPA-calculation using an octupole-octupole force, the two nuclear states based on these two configurations were calculated in perturbation theory including contributions up to third order in the particle-vibration coupling strength in the matrix element. The (t,p) reaction to the final states in ^{210}Pb can go directly either to the two-particle component or through the ground state correlations in ^{208}Pb to the $3p-1h$ component. Since the octupole state of ^{208}Pb is strongly collective, there is also a large possibility for the reaction to go by first exciting $^{208}\text{Pb}(3^-)$ and then transferring to the final state or by first transferring to the ground state of ^{210}Pb and then exciting the final state inelastically. These routes were also considered by performing the reaction calculation in CCBA. The final result shows a much improved agreement compared to the result obtained

if only the 2p component is allowed to be excited directly. Thus both ground state correlations and coupled channels effects are important for a correct reaction description.

Another analysis using pairing wave functions is presented in paper XI for the reaction $^{204}\text{Pb}(t,p)^{206}\text{Pb}$. The low-lying states of ^{206}Pb are built mainly of two-hole components with respect to $^{208}\text{Pb}(\text{gs})$. These states are calculated with multipole-pairing forces. The agreement between experimental and theoretical DWBA cross sections is good. However, since the pairing and octupole-octupole forces produce strongly correlated entities one can consider these states as elementary modes of excitation which can couple to form higher excited states. Thus there should be a region in the spectrum of ^{206}Pb having states of 1p-3h character (formed by coupling states in ^{206}Pb with excitations of ^{208}Pb) and still higher a region of states of 2p-4h type ($^{206}\text{Pb}(\text{gs}) \otimes ^{206}\text{Pb}(\text{gs}) \otimes ^{210}\text{Pb}(\lambda)$) etc. This picture is well born out by the $^{204}\text{Pb}(t,p)^{206}\text{Pb}$ reaction. It is also confirmed in the DWBA analysis described in paper XI. An even better agreement for the 1p-3h states than that obtained would probably be the result if an analysis similar to the one in paper III for the $^{208}\text{Pb}(t,p)^{210}\text{Pb}(3^-)$ reaction were performed.

Evidence for multipole-pairing correlations has also been obtained for deformed nuclei (papers VI and VII). In the (p,t) reactions to some Th and U isotopes low-lying monopole states have been excited by as much as 15% of the ground state cross section. This is consi-

derably more than what is expected for a two-quasiparticle excitation. On the other hand, these states were not excited in the (t,p) reaction. The quadrupole pairing acts in such a way that the pairing matrix element between orbits having quadrupole moments of different signs can be very small. Thus oblate and prolate sets of orbits can be very much decoupled from each other, and one set may give rise to a coherent ground state and the other one to a coherent (in the large components) excited state. If the excited state is built on orbits mainly above or mainly below the Fermi energy it will be populated very differently in two-nucleon stripping and pick-up reactions. In paper VII is presented a calculation which shows this effect very nicely for ^{234}U . There is therefore reason to interpret the above mentioned low-lying monopole states as being determined by the interplay between monopole and quadrupole pairing forces.

IV. Two-nucleon transfer and the coexistence model

Since the spectra of nuclei around ^{16}O and ^{40}Ca to a large extent can be interpreted as a set of rotational bands, it has been argued, that the states of these nuclei should be described containing both spherical and deformed components⁹⁾. This coexistence model could satisfactorily explain the existence of low-lying 0^+ states. Rather complete (p,t) and (t,p) data exist for the even Ca isotopes. In paper IV an analysis of these was carried out. Coexistence model wave functions were determined by varying their amplitudes until a reasonable fit to the data was obtained. Also other wave

functions were tested. The main conclusion from the last analysis was that the $0f_{7/2}$ and $1p_{3/2}$ shells are insufficient for a description of the Ca-isotopes and that other configurations are important. A test calculation also showed how the empirical factor for the absolute cross section normalization ($N \sim 25$) arises when the zero-range approximation is used. Firstly, as shown by Bayman¹⁰⁾, the finite-range corrections are sizeable. Secondly, one should take into account all the distant configurations, which, although being very small, add up so as to increase the absolute magnitude of the form factor by as much as a factor 1.7.

It has recently been shown¹¹⁾ that the coexistence model calculated by Gerace and Green⁹⁾ for ^{40}Ca gives cross sections in strong disagreement with the results of the reaction $^{42}\text{Ca}(p,t)^{40}\text{Ca}$. This has motivated the investigation presented in paper X. The (p,t) reaction is very selective for populating different configurations. Thus, a $(1s_{1/2})_0^{2+}$ state has a cross section much larger than a pure configuration $(0d_{3/2})_0^{2+}$. This is so partly because of the different amount of relative S-motion in the configurations and partly because the $1s_{1/2}$ orbit extends to larger radii than does the $d_{3/2}$ state. The main reason for the above mentioned discrepancy seems to be the retaining of axial symmetry when the deformed orbits are calculated. The hole orbit in the deformed components has $K=3/2$ and the $1s_{1/2}$ state can not be mixed in. If, however, a γ -deformation is introduced, K is not a good quantum number any more, and the hole pick-up can now receive a considerable contribution from the $1s_{1/2}$ orbit. It is shown in paper

X that with reasonable deformations the disagreement for the cross section to the first excited 0^+ state in ^{40}Ca can be reduced to a factor of two, which should be considered satisfactory taking all the uncertainties connected with the DWBA calculations into account.

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